Lecture 10: Learning (Coarse)-Correlated Equilibria in General-Sum Games

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October 16, 2025

Abstract

We introduce the equilibrium concepts of correlated, coarse correlated, and more generally Φ-equilibria. In general-sum games, we show that if all players use no-internal, no-external and more generally no-Φ-regret learning algorithms, the joint empirical distribution of play respectively converges to the sets of correlated, coarse-correlated and Φ-equilibria.

Disclaimer: These lecture notes are preliminary notes that will evolve and will be updated over time, this is the first iteration of the course. The treatment is not comprehensive but focuses on some of the main ideas with accompanying proofs. Research on the topic is also still very active.

1 Coarse Correlated and Correlated Equilibria

Throughout this section, consider a finite normal-form game $\Gamma = (\mathcal{I}, \{\mathcal{A}_i\}_{i\in\mathcal{I}}, \{u_i\}_{i\in\mathcal{I}})$. For each agent $i \in \mathcal{I}$, we denote by $\mathcal{X}_i := \Delta(\mathcal{A}_i)$ the set of mixed strategies.

1.1 Hardness of Nash equilibrium computation

Besides special cases such as the zero-sum or potential settings discussed in the previous lectures, computing a Nash equilibrium is computationally hard in general.[1] Papadimitriou (1994) introduced the PPAD complexity class (PPAD stands for Polynomial Parity Arguments on Directed graphs), showing that computing Brouwer fixed points is PPAD-complete and crystallizing why fixed-point–guaranteed solutions may be intractable in general. Building on this framework, Daskalakis et al. (2009a) proved that computing a (mixed) Nash equilibrium in games with three or more players is PPAD-complete, relating Nash search to Brouwer's fixed point computation.[2] The two-player case was later settled by Chen and Deng (2006), who showed that computing a bimatrix Nash equilibrium is also PPAD-complete, eliminating hopes for a general polynomial-time algorithm even in the bimatrix setting.

1.2 Coarse correlated equilibrium

Recall that a mixed strategy Nash equilibrium is a strategy profile $(x_1, \dots, x_n) \in \prod_{k=1}^N \Delta(\mathcal{A}_k)$ such that no unilateral deviation is profitable for any player, i.e.,

$$\forall i \in \mathcal{I}, \quad \forall a_i' \in \mathcal{A}_i\,, \quad u_i(a_i', x_{-i}) \leq u_i(x_i, x_{-i})\,. \tag{1}$$

Note in particular that players randomize their strategies *independently*. The probability of playing a joint action $a = (a_1, \dots, a_N) \in \prod_{k=1}^N \mathcal{A}_k$ is given by the product $\prod_{k=1}^N x_{k,a_k}$ of the probabilities of each agent $k \in \mathcal{I}$ playing action a_k.

Coarse correlated equilibria relax this definition and allow for correlated strategies, i.e. strategies $x \in \Delta(\prod_{k=1}^N \mathcal{A}_k)$ (rather than only independent ones in $\prod_{k=1}^N \Delta(\mathcal{A}_k)$).

[1] We will not cover this topic in details in this course, we only provide a brief account without formal definitions and statements.

[2] For a shorter, less technical and more accessible exposition, see Daskalakis et al. (2009b) for the main ideas.

Definition 1 (Coarse correlated equilibrium (Moulin and Vial, 1978)**).** *A coarse correlated equilibrium (CCE) is a correlated strategy $x \in \Delta(\prod_{k=1}^{N} \mathcal{A}_k)$ such that*

$$\mathbb{E}_{(a_i,a_{-i})\sim x}\left[u_i(a_i', a_{-i})\right] \;\leq\; \mathbb{E}_{(a_i,a_{-i})\sim x}[u_i(a_i, a_{-i})] \qquad \forall i \in \mathcal{I},\, a_i' \in \mathcal{A}_i. \tag{2}$$

Remark 2. A Nash equilibrium is a CCE x which is a product distribution. As a consequence, the set of CCEs is a superset of the set of Nash equilibria. In particular, a coarse correlated equilibrium always exists (in any game).

Proposition 3. *The set of CCEs is a convex polytope.*

Proof. A CCE can be described by a finite dimensional vector $x \in \mathbb{R}^{|\mathcal{A}|}$ whose entries $x_{a_1,\cdots,a_n}$ (for $(a_1,\ldots,a_N) \in \mathcal{A}$ satisfy:

$$x_{a_1,\ldots,a_N} \geq 0, \quad \forall (a_1,\ldots,a_N) \in \mathcal{A}\,, \qquad \sum_{a_1\in\mathcal{A}_1,\cdots,a_N\in\mathcal{A}_N} x_{a_1,\ldots,a_N} = 1\,, \tag{3}$$

$$\sum_{a_1\in\mathcal{A}_1,\cdots,a_n\in\mathcal{A}_n} x_{a_1,\ldots,a_N}\, u_i(a_i', a_{-i}) \leq \sum_{a_1\in\mathcal{A}_1,\cdots,a_N\in\mathcal{A}_n} x_{a_1,\ldots,a_N}\, u_i(a_i, a_{-i}) \quad \forall\, i \in \mathcal{I},\; a_i' \in \mathcal{A}_i\,, \tag{4}$$

where the last constraints follow from expanding the expectation in the inequalities defining CCEs (see Definition 1). It follows that the set of CCEs is the intersection of a finite set of linear constraints. Therefore it is a convex polytope. □

Remark 4 (Computation)**.** The above proof also shows that computing a CCE can be recast as solving an optimization problem where the variables are the entries of the probability distribution x. The number of constraints is polynomial in the size of the payoff table, and linear programming can be used to compute and even optimize over the set of CCEs in time polynomial in the product $\prod_{k=1}^{N} |\mathcal{A}_k|$.

In contrast, the set of Nash equilibria is not guaranteed to be convex in general beyond zero-sum games, and can be quite complex topologically, see e.g. Kohlberg and Mertens (1986).

CCEs may contain correlated strategies that assign positive probability only to strictly dominated strategies (Viossat and Zapechelnyuk, 2013). In contrast, correlated equilibria cannot be supported on dominated strategies.

1.3 Correlated equilibrium

Definition 5 (Correlated equilibrium (Aumann, 1974)**).** *A correlated equilibrium (CE) is a correlated strategy $x \in \Delta(\prod_{k=1}^{N} \mathcal{A}_k)$ such that*

$$\mathbb{E}_{(a_i,a_{-i})\sim x}[u_i(\phi_i(a_i), a_{-i})] \;\leq\; \mathbb{E}_{(a_i,a_{-i})\sim x}[u_i(a_i, a_{-i})] \qquad \forall i \in \mathcal{I},\, \forall\, \phi_i : \mathcal{A}_i \to \mathcal{A}_i. \tag{5}$$

Remark 6. A similar definition with a conditioning formulation can also be found: *After* I see my private recommendation, following it should be at least as good as switching to any fixed action.

Proposition 7. *The set of CEs is a convex polytope.*

Proof. Same as for CCEs. □

Remark 8. For more about existence of correlated equilibria, see Hart and Schmeidler (1989) who provide a (non-fixed point) proof of the nonemptiness of the set of correlated equilibria.

Remark 9. Similarly to CCEs, CEs can be computed in polynomial time using linear programming.

Remark 10. NE $\subset$ CE $\subset$ CCE, see Fig. 1.

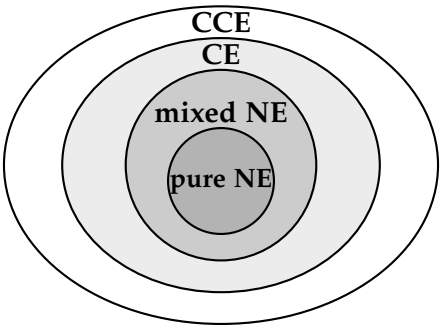

Figure 1: Equilibrium concepts. Pure NE: pure Nash equilibrium (need not always exist, difficult to compute in general); Mixed NE: Mixed Nash equilibrium (always guaranteed to exist, hard to compute in general); CE: correlated equilibrium (easy to compute, e.g. via no-internal regret algorithms), CCE: coarse correlated equilibrium (easy to learn, e.g. via no-external regret algorithms).

1.4 Interpretation of correlated play in games

In this section, we provide an interpretation of correlated play in games using what is commonly used a correlation devise or a mediator.

A correlation device is a trusted "referee" (or mediator) that suggests actions to players. It cannot force any player to follow the suggestion, but every player knows the rule it uses to make suggestions. The device has a public distribution x which is a probability distribution for drawing a full action profile. Before each play of the game, the device draws $(a_1, \ldots, c_N) \sim x$ and privately tells player i only their recommendation. Each player can either follow their recommendation or switch to some other action.

We comment now on the difference between CE and CCE:

- For CCE, players decide in advance whether they will commit to "follow whatever the device tells me", before seeing any recommendation. Profitable deviations to check are hence alternative actions which do not depend on the message.
- For CE, players can wait to see the recommendation and then decide whether to follow it. Now the deviation can depend on the message ("if the device says L, switch to R; otherwise follow").

Both CE and CCE allow players to coordinate using shared random advice. CE gives players more flexibility in how to deviate (after seeing the message). CCE is therefore a weaker equilibrium notion (easier to satisfy), while CE is stronger.

For an example illustrating the gap between CE and CCE, see for instance example 5.7 in Ratliff (2021).

1.5 Example: Traffic light

Two players choose go or stop. Payoffs are as follows:

	stop	go
stop	$(0,0)$	$(0,1)$
go	$(1,0)$	$(-5,-5)$

Mediator (traffic light). Let x put probability $\frac{1}{2}$ on $(\mathsf{go}, \mathsf{stop})$ and $\frac{1}{2}$ on $(\mathsf{stop}, \mathsf{go})$. This is *public knowledge*, but each player privately sees only their own recommendation.

Check CE (after seeing the message).

- If Player 1 is told go, she infers Player 2 was told stop. Following: $u_1(\mathsf{go}, \mathsf{stop}) = 1$. Deviate to stop: $u_1(\mathsf{stop}, \mathsf{stop}) = 0$. So following is better.
- If Player 1 is told stop, she infers Player 2 was told go. Following: $u_1(\mathsf{stop}, \mathsf{go}) = 0$. Deviate to go: $u_1(\mathsf{go}, \mathsf{go}) = -5$. So following is better.

By symmetry for Player 2, x is a **CE**.

Check CCE (commit before seeing the message). For Player 1:

$$\mathbb{E}[\text{Follow}] = \tfrac{1}{2}\cdot 1 + \tfrac{1}{2}\cdot 0 = 0.5, \qquad \mathbb{E}[\text{Always go}] = \tfrac{1}{2}\cdot 1 + 0\cdot(-5) = 0.5, \qquad \mathbb{E}[\text{Always stop}] = 0.$$

Thus $0.5 \geq 0$ and $0.5 \geq 0.5$, so following weakly dominates any fixed deviation; x is a **CCE**. (Again, symmetry gives the same for Player 2.)

Not a mixed Nash. The distribution x never recommends $(\mathsf{go}, \mathsf{go})$ or $(\mathsf{stop}, \mathsf{stop})$, so x is not a product distribution; hence it cannot be a mixed Nash equilibrium (which is independent by definition). Independence cannot produce "never both go, never both stop" unless it collapses to a single pure outcome.

Takeaway. The distribution is public (everyone knows how the light is drawn), but messages are private (each driver only sees their own light). CE checks best responses *after* seeing the private message; CCE checks that committing to follow *before* seeing it beats any fixed always-do-a_i' rule. This traffic-light x satisfies both.

1.6 Φ-equilibrium

Let Φ_i be a finite subset of linear maps $\phi_i : \Delta(\mathcal{A}_i) \to \Delta(\mathcal{A}_i)$. A linear map ϕ_i extends to $\phi_i : \Delta(\prod_{k=1}^N \mathcal{A}_k) \to \Delta(\prod_{k=1}^N \mathcal{A}_k)$ as follows for any $q \in \Delta(\prod_{k=1}^N \mathcal{A}_k), b_i \in \mathcal{A}_i, a_{-i} \in \mathcal{A}_{-i}$:

$$\begin{aligned}
\phi_i(q)(b_i, a_{-i}) &\equiv \phi_i\Big(\big(q(a_i, a_{-i})\big)_{a_i \in \mathcal{A}_i}\Big)(b_i) \\
&= \phi_i\Big(\sum_{a_i \in \mathcal{A}_i} q(a_i, a_{-i})\, \delta_{a_i}\Big)(b_i) \\
&= \sum_{a_i \in \mathcal{A}_i} q(a_i, a_{-i})\, \phi_i(\delta_{a_i})(b_i). \qquad (6)
\end{aligned}$$

A distribution $q \in \Delta(A_1 \times \cdots \times A_n)$ is said to be *independent* if and only if $q = q_1 \otimes \cdots \otimes q_n$ with $q_i \in \Delta(\mathcal{A}_i)$.

Definition 11 (Φ-equilibrium). *Given $\Phi = (\Phi_i)_{1 \leq i \leq N}$, a distribution $q \in \Delta(\prod_{i=1}^N \mathcal{A}_i)$ is called a Φ-equilibrium if and only if:*

$$\forall i \in \mathcal{I}, \quad \forall \phi_i \in \Phi_i, \quad u_i(\phi_i(q)) \leq u_i(q)\,. \qquad (7)$$

We recover the equilibrium concepts introduced in the previous sections:

- Coarse correlated equilibria are Φ-equilibria with $\Phi = \Phi_{\text{EXT}}$ equal to the set of constant transformations, i.e., $\Phi_{\text{EXT}} = \{\phi_x \mid x \in \mathcal{A}\}$ where $\phi_x(a) = x$ for all $a \in \mathcal{A}$.
- Correlated equilibria are Φ-equilibria with $\Phi = \Phi_{\text{INT}} = \{\phi_{ab} \mid a \neq b \in \mathcal{A}\}$, where

$$\big(\phi_{ab}(q)\big)_c = \begin{cases} q_c & \text{if } c \neq a, b, \\ 0 & \text{if } c = a, \\ q_a + q_b & \text{if } c = b. \end{cases} \qquad (4)$$

Lemma 12. *The set of Φ-equilibria is convex.*

Proof. The proof is left as an exercise. □

Remark 13. A Nash equilibrium is an independent Φ-equilibrium. The set of Φ-equilibria contains the set of NE. Since the set of Φ-equilibria is convex, it also contains the convex hull of the set of NE (since the convex hull is the smallest convex set containing the points). However the convex hull of the set of NE need not contain even the smallest set of Φ-equilibria (set of correlated equilibria). See Greenwald and Jafari (2003) for more details.

2 Learning (Coarse) Correlated Equilibria

Recall from the previous lecture on learning Nash equilibria in zero-sum games via online learning the following result proved in Freund and Schapire (1996):

> *In two-player zero-sum games, if each player plays using a no-external regret algorithm, then the empirical distribution of play converges to the set of minimax equilibria.*

In this section, we state mirroring (more general) statements for CCEs, CEs and beyond.

2.1 Convergence to the set of CCEs and CEs

We first state a result for CCEs which appeared in Hannan (1957).

Theorem 14. *When all agents in a multiplayer general-sum normal-form game play using a no-external-regret learning algorithm, their empirical distribution of play converges to the set of coarse correlated equilibria of the game.*

The following result regarding learning CEs appeared in a number of works in the literature (Foster and Vohra, 1993; Fudenberg and Levine, 1999; Hart and Mas-Colell, 2000, 2001).

Theorem 15. *When all agents in a multiplayer general-sum normal-form game play using a no-internal-regret learning algorithm, then the empirical distribution of play converges to the set of correlated equilibria of the game.*

2.2 No-Φ-regret learning and convergence to the set of Φ-equilibria

2.2.1 Refresher on no-Φ-regret learning

Recall the repeated (convex) game setting:

- At each time step t, the agent chooses an action $x_t \in \mathcal{A}$.
- At the same time, the environment (forces external to the agent) chooses a convex loss function $\ell_t \in L$ (loss is just a negative payoff).
- The agent observes ℓ_t and pays $\ell_t(x_t)$.

In our game setting, the action space is $\Delta(\mathcal{A}_i)$. The rest of this section is also valid for any convex and compact subset of $\mathbb{R}^d$. The set L includes convex loss functions with bounded subgradients.

Learning algorithm. This is an algorithm taking as input a sequence of loss functions ℓ_t and producing a sequence of actions x_t. Action x_t may depend on $\ell_1, \ldots, \ell_{t-1}$, but not on ℓ_t or later loss functions. The learner's objective is to minimize its cumulative loss $\sum_{t=1}^{T} \ell_t(x_t)$. The **regret** evaluates the performance of a learning algorithm against a given sequence. The simplest type

is *external regret*, defined by

$$\mathrm{Reg}_{\mathrm{EXT}}^T \;=\; \sup_{x\in\mathcal{A}} \sum_{t=1}^{T} \big(\ell_t(x_t) - \ell_t(x)\big).$$

The external regret is the difference between the actual loss achieved and the smallest possible loss that could have been achieved on the sequence ℓ_t by playing a fixed $x \in \mathcal{A}$.

An algorithm exhibits no-external-regret for feasible region $\mathcal{A}$ and set L if there is a function $f(T, \mathcal{A}, L)$ which is $o(T)$ for any fixed $\mathcal{A}$ and L, such that for all $x \in \mathcal{A}$, $t \geq 1$,

$$\sum_{t=1}^{T} \ell_t(x_t) \;\leq\; \sum_{t=1}^{T} \ell_t(x) \;+\; f(T, \mathcal{A}, L).$$

More generally, an agent can consider replacing its sequence $a_1 \dots a_T$ with $\phi(a_1) \dots \phi(a_T)$, where ϕ is some transformation mapping $\mathcal{A}$ to itself. If Φ is a set of such transformations, we define an algorithm's Φ-regret as

$$\Phi\text{-}\mathrm{Reg}^T \;=\; \sup_{\phi\in\Phi} \sum_{t=1}^{T} \big(\ell_t(x_t) - \ell_t(\phi(x_t))\big),$$

and we say that it exhibits no-Φ-regret if it satisfies for all $\phi \in \Phi$, $t \geq 1$,

$$\sum_{t=1}^{T} \ell_t(x_t) \;\leq\; \sum_{t=1}^{T} \ell_t\big(\phi(x_t)\big) \;+\; g(T, \mathcal{A}, L, \Phi),$$

where $g(T, \mathcal{A}, L, \Phi)$ is $o(T)$ for any fixed $\mathcal{A}, L$, and Φ.

Recall that we recover external and internal regret by setting the set Φ of linear maps as follows:

- External regret is Φ-regret with $\Phi = \Phi_{\mathrm{EXT}}$ (set of constant transformations).
- Internal regret is Φ-regret with $\Phi = \Phi_{\mathrm{INT}}$.

2.2.2 Main theorem

Theorem 16. *Let $x_1^t, \dots, x_N^t$ be the strategies played by the players at any time t, and let Φ-Reg_i^t denote the internal regret incurred by player i up to time t. Consider now the average correlated distribution of play up to any time T, i.e. the distribution $\bar{x}^T$ that selects a time $\bar{t}$ uniformly at random from the set $\{1, \dots, T\}$, and then selects actions $(a_1, \dots, a_n)$ independently according to the $x_i^{\bar{t}}$:*

$$\bar{x}^T \;:=\; \frac{1}{T} \sum_{t=1}^{T} x_1^t \otimes \cdots \otimes x_n^t. \tag{8}$$

Then, this distribution satisfies the inequality

$$\max_{\phi\in\Phi} \; \mathop{\mathbf{E}}_{a\sim\bar{x}^T} \big[\, u_i(\phi(a_i), a_{-i}) - u_i(a_i, a_{-i}) \,\big] \;\leq\; \frac{\Phi\text{-}Reg_i^T}{T}\,. \tag{9}$$

Proof. Let $\phi \in \Phi$. Using the definition of $\bar{x}^T$, linearity of ϕ, of the expectation and multi-

linearity of the utilities, we can write:

$$\begin{aligned}
\mathop{\mathbf{E}}_{a\sim\bar{x}^T}[u_i(\phi(a_i),a_{-i})-u_i(a_i,a_{-i})] &= \frac{1}{T}\sum_{t=1}^{T}\mathop{\mathbf{E}}_{a\sim x_1^t\otimes\cdots\otimes x_n^t}\left[\,u_i(\phi(a_i),a_{-i})-u_i(a_i,a_{-i})\,\right]\\
&= \frac{1}{T}\sum_{t=1}^{T}\left(u_i\left(\mathop{\mathbf{E}}_{a_i\sim x_i^t}[\phi(a_i)],\mathop{\mathbf{E}}_{a_{-i}\sim x_{-i}^t}[a_{-i}]\right)\right)-u_i\Big(\mathop{\mathbf{E}}_{a_i\sim x_i^t}[a_i],\mathop{\mathbf{E}}_{a_{-i}\sim x_{-i}^{(t)}}[a_{-i}]\Big)\Big)\\
&= \frac{1}{T}\sum_{t=1}^{T}\left(u_i(\phi(x_i^t),\,x_{-i}^t)-u_i(x_i^t,\,x_{-i}^t)\right).
\end{aligned}$$

Taking now a maximum over $\phi\in\Phi$, and recognizing the definition of Φ-regret yields the desired inequality. □

We discuss a few special cases recovering the main theorems for CCEs and CEs:

- When the set of transformations Φ is the set of all constant transformations, the previous result implies convergence to the set of coarse correlated equilibria.
- When the set of transformations Φ is the set of all arbitrary mappings $\phi:\mathcal{A}_i\to\mathcal{A}_i$, the previous result implies convergence to the set of coarse correlated equilibria.

Define the empirical distribution z_t of play through time t as follows:

$$\forall a_i\in\mathcal{A}_i, a_{-i}\in\mathcal{A}_{-i}=\prod_{j\neq i}\mathcal{A}_j,\quad z_t(a_i,a_{-i}) \;=\; \frac{1}{t}\sum_{\tau=1}^{t}\mathbf{1}_{\{a_{i,\tau}=a_i\}}\,\mathbf{1}_{\{a_{-i,\tau}=a_{-i}\}}\,, \tag{10}$$

where $\mathbf{1}_{\{\cdot\}}$ denotes the indicator function, which equals 1 whenever the condition in braces holds and 0 otherwise.

Theorem 17 (Greenwald and Jafari (2003))**.** *If all players i play according to a Φ_i-no-regret algorithm, then the joint empirical distribution of play z_t defined in (10) converges almost surely to the set of Φ-equilibria.*

2.3 Extension to continuous games

Extensions of the results we discussed to continuous games with compact strategy sets have been investigated in the literature (Stoltz and Lugosi, 2007; Gordon et al., 2008; Hazan and Kale, 2007).

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